## PROSIDING

SEMINAR NASIONAL<br>MATEMATIKA\&<br>PENDIDIKAN MATEMATIKA



## Tema :

"Peran Matematika dan Pembelajarannya dalam Meningkatkan Daya Saing Bangsa"

PROGRAM STUDI PENDIDIKAN MATEMATIKA FKIP UNIVERSITAS JEMBER

$$
2011
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## PROGRAM STUDI PENDIDIKAN MATEMATIKA FKIP UNIVERSITAS JEMBER

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# Super Edge-antimagic Total Labeling of Stair Graph 

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#### Abstract

An $(a, d)$-edge-antimagic total labeling of $G$ is a one-to-one mapping $f$ taking the vertices and edges onto $\{1,2,3, \ldots, p+q\}$. So that the edge-weights $w(u v)=$ $f(u)+f(v)+f(u v), u v \in E(G)$, form an arithmetic progression $\{a, a+d, a+$ $2 d, \ldots, a+(q-1) d\}$, where $a>0$ and $d \geq 0$ are two fixed integers, and form an arithmetic sequence with first term $a$ and common difference $d$. Such a graph $G$ is called super if the smallest possible labels appear on the vertices. In this paper we survey what is known about super edge-antimagic total labelings properties of disconnected Stair graph $\left(S t_{n}\right)$.


Keywords : (a,d)-edge-antimagic total labeling, super ( $a, d$ )-edge-antimagic total labeling, Stair graph.

## 1 Introduction

In mathematics and computer science, graph theory a content to study of graphs, mathematical structures used to model pairwise relations between objects from a certain collection. A "graph" in this context refers to a collection of vertices or 'nodes' and a collection of edges that connect pairs of vertices. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another. Graphs are one of the prime objects of study in discrete mathematics.
A labeling of a graph is any mapping that sends some set of graph elements to a set of positive integers. If the domain is the vertex-set or the edge-set, the labelings are called, respectively, vertex labelings or edge labelings. Moreover, if the domain is $V(G) \cup E(G)$ then the labelings are called total labelings. We define the edge-weight of an edge $u v \in E(G)$ under a total labeling to be the sum of the vertex labels corresponding to vertices $u, v$ and edge label corresponding to edge $u v$. If such a labeling exists then $G$ is said to be an $(a, d)$-edge-antimagic total graph. Such a graph $G$ is called super if the smallest possible labels appear on the vertices. Thus, a super $(a, d)$-edge-antimagic total graph is a graph that admits a super $(a, d)$-edge-antimagic total labeling.

Definitions of ( $a, d$ )-EAT labeling and super (a,d)-EAT labeling were introduced by Simanjuntak at al [25]. These labelings are natural extensions of the notion of edgemagic labeling, dened by Kotzig and Rosa [17], where edge-magic labeling is called magic valuation, and the notion of super edge-magic labeling, which was defined by Enomoto, Llado, Nakamigawa and Ringel [11]. in [19], is natural extension of the notion of edge-magic labeling defined by Kotzig and Rosa [2] (see also [17], [18], [12] and [22]). The super ( $a, d$ )-edge-antimagic total labeling is natural extension of the notion of super edge-magic labeling which was defined by Enomoto et al. in [14].

In this paper we investigate the existence of super $(a, d)$-edge-antimagic total labelings of Stair graph, denoted by $S t_{n}$.

## 2 Research Methods and Techniques

The research techniques are as follows: (1) calculate the number of vertex $p$ and size $q$ on the graph $S t_{n} ;(2)$ determine the upper bound for values of $d ;(3)$ determine the $E A V L$ (edge-antimagic vertex labeling) of $S t_{n} ;(4)$ if the label of $E A V L$ is expandable, then we continue to determine the bijective function of $E A V L$; (5) label the graph $S t_{n}$ with $S E A T L$ (super-edge antimagic total labeling) with feasible values of $d$ and (6) determine the bijective function of super-edge antimagic total labeling of graph $S t_{n}$.

## 3 Some Useful Lemmas

We start this section by a necessary condition for a graph to be super (a,d)-edge antimagic total, providing a least upper bound for feasible values of $d$.

Lemma 1 If $a(p, q)$-graph is super $(a, d)$-edge antimagic total then $d \leq \frac{2 p+q-5}{q-1}$.
Proof. Assume that a $(p, q)$-graph has a super $(a, d)$-edge antimagic total labeling $f: V(G) \cup E(G) \rightarrow\{1,2, \ldots, p+q\}$. The minimum possible edge-weight in the labeling $f$ is at least $1+2+p+1=p+4$. Thus, $a \geq p+4$. On the other hand, the maximum possible edge-weight is at most $(p-1)+p+(p+q)=3 p+q-1$. So we obtain $a+(q-1) d \leq 3 p+q-1$ which gives the desired upper bound for the difference $d$.

## 4 Stair Graph

Let's key n is the numbers of each stair. Stair graph denoted by $S t_{n}$ is a connected graph with vertex set $V\left(S t_{n}\right)=\left\{x_{i}, y_{i}, z_{j}, q_{k} ; 1 \leq i \leq n, 1 \leq j \leq 2 n+2,1 \leq k \leq 4 n\right\}$, and $E\left(S t_{n}\right)=\left\{x_{i} y_{i}, x_{i} z_{2 i \pm 1}, y_{i} z_{2 i+2}, y_{i} z_{2 i}, y_{i} q_{4 i-3}, y_{i} q_{4 i-2}, x_{i} q_{4 i-1}, x_{i} q_{4 i} ; 1 \leq i \leq n\right\} \cup$ $\left\{q_{i} q_{i+1} ; i\right.$ odd, $\left.1 \leq i \leq 4 n-1\right\} \cup\left\{z_{i} z_{i+1} ; i\right.$ odd, $\left.1 \leq i \leq 2 n+1\right\} \cup\left\{z_{i} q_{2 i-1}, z_{i} q_{2 i} ; i\right.$ odd, $1 \leq$ $i \leq 2 n-1\} \cup\left\{z_{i} q_{2 i-4}, z_{i} q_{2 i-5} ; i\right.$ even, $\left.4 \leq i \leq 2 n+2\right\}$ Thus $\left|V\left(S t_{n}\right)\right|=p=8 n+2$ and $\left|E\left(S t_{n}\right)\right|=q=16 n+1$.

If Stair graph, has a super $(a, d)$-edge-antimagic total labeling then, for $p=8 n+2$ and $q=16 n+1$, it follows from Lemma 1 , that the upper bound of $d$ is $d \leq 2$ or
$d \in\{0,1,2\}$.

$$
\begin{aligned}
d & \leq \frac{2 p+q-5}{q-1} \\
& =\frac{2(8 n+2)+16 n+1-5}{16 n+1-1} \\
& =\frac{16 n+4+16 n-4}{16 n} \\
& =\frac{32 n}{16 n} \\
& \leq 2
\end{aligned}
$$

The following lemma describes an ( $a, 1$ )-edge-antimagic vertex labeling for Stair Graph.

Theorem 1 If $n \geq 2$, then the Stair Graph connected $S t_{n}$ has an ( $a, 1$ )-edge-antimagic vertex labeling.

Proof. Define the vertex labeling $\alpha_{1}: V\left(S t_{n}\right) \rightarrow\{1,2, \ldots, 8 n+2\}$ in the following way:

$$
\begin{gathered}
\alpha_{1}\left(x_{i}\right)=8 i-3, \text { for } 1 \leq i \leq n \\
\alpha_{1}\left(y_{i}\right)=8 i-2 \text {, for } 1 \leq i \leq n \\
\alpha_{1}\left(z_{j}\right)=4 j-3-\frac{\left((-1)^{j}+1\right) 3}{2}, \text { for } 1 \leq j \leq 2 n+2, \text { any } l \\
\alpha_{1}\left(q_{k}\right)=2 k+\frac{\left((-1)^{k+1}+1\right)}{2}, \text { for } 1 \leq k \leq 4 n, \text { any } l
\end{gathered}
$$

The vertex labeling $\alpha_{1}$ is a bijective function. The edge-weights of $S t_{n}$, under the labeling $\alpha_{1}$, constitute the following sets :

$$
\begin{gathered}
w_{\alpha_{1}}^{1}\left(x_{i} y_{i}\right)=16 i-5, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{2}\left(x_{i} z_{2 i+1}\right)=16 i-2, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{3}\left(x_{i} z_{2 i-1}\right)=16 i-10, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{4}\left(x_{i} q_{4} i\right)=16 i-3, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{5}\left(x_{i} q_{4 i-1}\right)=16 i-4, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{6}\left(y_{i} z_{2 i+2}\right)=16 i, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{7}\left(y_{i} z_{2 i}\right)=16 i-8, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{8}\left(y_{i} q_{4 i-2}\right)=16 i-6, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{9}\left(y_{i} q_{4 i-3}\right)=16 i-7, \text { for } 1 \leq i \leq n \\
w_{\alpha_{1}}^{10}\left(z_{i} q_{2 i-1}\right)=8 i-4, \text { for } 1 \leq i \leq 2 n-1, \text { odd }
\end{gathered}
$$

$$
\begin{aligned}
w_{\alpha_{1}}^{11}\left(z_{i} q_{2 i}\right) & =8 i-3, \text { for } 1 \leq i \leq 2 n-1, \text { odd } \\
w_{\alpha_{1}}^{13}\left(z_{i} q_{2 i-4}\right) & =8 i-14, \text { for } 4 \leq i \leq 2 n+2, \text { even } \\
w_{\alpha_{1}}^{13}\left(z_{i} q_{2 i-5}\right) & =8 i-15, \text { for } 4 \leq i \leq 2 n+2, \text { even } \\
w_{\alpha_{1}}^{14}\left(q_{i} q_{i+1}\right) & =4 i+3, \text { for } 1 \leq i \leq 4 n-1, \text { odd } \\
w_{\alpha_{1}}^{15}\left(z_{i} z_{i+1}\right) & =8 i-5, \text { for } 1 \leq i \leq 2 n+1, \text { odd }
\end{aligned}
$$

It is not difficult to see that the set $\bigcup_{t=1}^{15} w_{\alpha_{1}}^{t}=\{3,4,5, \ldots, 16 n+3\}$ consists of consecutive integers. Thus $\alpha_{1}$ is a (3,1)-edge antimagic vertex labeling.

Theorem 2 If $n \geq 2$ then the graph $S t_{n}$ has a super $(24 n+6,0)$-edge-antimagic total labeling and a super ( $8 n+6,2$ )-edge-antimagic total labeling.

Proof. Case 1. for $d=0$
Label the vertices of $S t_{n}$ and label the edges with the following way.

$$
\begin{aligned}
\alpha_{2}\left(x_{i} y_{i}\right) & =24 n-16 i+11, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(x_{i} z_{2 i+1}\right) & =24 n-16 i+8, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(x_{i} z_{2 i-1}\right) & =24 n-16 i+16, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(x_{i} q_{4} i\right) & =24 n-16 i+9, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(x_{i} q_{4 i-1}\right) & =24 n-16 i+10, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(y_{i} z_{2 i+2}\right) & =24 n-16 i+6, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(y_{i} z_{2 i}\right) & =24 n-16 i+14, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(y_{i} q_{4 i-2}\right) & =24 n-16 i+12, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(y_{i} q_{4 i-3}\right) & =24 n-16 i+13, \text { for } 1 \leq i \leq n \\
\alpha_{2}\left(z_{i} q_{2 i-1}\right) & =24 n-8 i+10, \text { for } 1 \leq i \leq 2 n-1, \text { odd } \\
\alpha_{2}\left(z_{i} q_{2 i}\right) & =24 n-8 i+9, \text { for } 1 \leq i \leq 2 n-1, \text { odd } \\
\alpha_{2}\left(z_{i} q_{2 i-4}\right) & =24 n-8 i+20, \text { for } 4 \leq i \leq 2 n+2, \text { even } \\
\alpha_{2}\left(z_{i} q_{2 i-5}\right) & =24 n-8 i+21, \text { for } 4 \leq i \leq 2 n+2, \text { even } \\
\alpha_{2}\left(q_{i} q_{i+1}\right) & =24 n-4 i+3, \text { for } 1 \leq i \leq 4 n-1, \text { odd } \\
\alpha_{2}\left(z_{i} z_{i+1}\right) & =24 n-8 i+11, \text { for } 1 \leq i \leq 2 n+1, \text { odd }
\end{aligned}
$$

The total labeling $\alpha_{2}$ is a bijective function from $V\left(S t_{n}\right) \cup E\left(S t_{n}\right)$ onto the set $\{1,2,3, \ldots, 24 n+3\}$. The edge-weights of $S t_{n}$, under the labeling $\alpha_{2}$, constitute the sets

$$
\begin{aligned}
W_{\alpha_{2}}^{1} & =\left\{w_{\alpha_{2}}^{1}+\alpha_{2}\left(x_{i} y_{i}\right) ; \text { for } 1 \leq i \leq n\right\} \\
& =(16 i-5)+(24 n-16 i+11) \\
& =24 n+6 \\
W_{\alpha_{2}}^{2} & =\left\{w_{\alpha_{2}}^{2}+\alpha_{2}\left(x_{i} z_{2 i+1}\right) ; \text { for } 1 \leq i \leq n\right\}
\end{aligned}
$$

$$
\begin{aligned}
& =(16 i-2)+(24 n-16 i+8) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{3}=\left\{w_{\alpha_{2}}^{3}+\alpha_{2}\left(x_{i} z_{2 i-1}\right) \text {; for } 1 \leq i \leq n\right\} \\
& =(16 i-10)+(24 n-16 i+16) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{4}=\left\{w_{\alpha_{2}}^{4}+\alpha_{2}\left(x_{i} q_{4 i}\right) ; \text { for } 1 \leq i \leq n\right\} \\
& =(16 i-3)+(24 n-16 i+9) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{5}=\left\{w_{\alpha_{2}}^{5}+\alpha_{2}\left(x_{i} q_{4 i-1}\right) ; \text { for } 1 \leq i \leq n\right\} \\
& =(16 i-4)+(24 n-16 i+10) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{6}=\left\{w_{\alpha_{2}}^{6}+\alpha_{2}\left(y_{i} z_{2 i+2}\right) ; \text { for } 1 \leq i \leq n\right\} \\
& =(16 i)+(24 n-16 i+6) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{7}=\left\{w_{\alpha_{2}}^{7}+\alpha_{2}\left(y_{i} z_{2 i}\right) ; \text { for } 1 \leq i \leq n\right\} \\
& =(16 i-8)+(24 n-16 i+14) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{8}=\left\{w_{\alpha_{2}}^{8}+\alpha_{2}\left(y_{i} q_{4 i-2}\right) ; \text { for } 1 \leq i \leq n\right\} \\
& =(16 i-6)+(24 n-16 i+12) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{9}=\left\{w_{\alpha_{2}}^{9}+\alpha_{2}\left(y_{i} q_{4 i-3}\right) ; \text { for } 1 \leq i \leq n\right\} \\
& =(16 i-7)+(24 n-16 i+13) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{10}=\left\{w_{\alpha_{2}}^{10}+\alpha_{2}\left(z_{i} q_{2 i-1}\right) ; \text { for } 1 \leq i \leq 2 n-1 \text { odd }\right\} \\
& =(8 i-4)+(24 n-8 i+10) \\
& =24 n+6 \\
& W_{\alpha_{2}}^{11}=\left\{w_{\alpha_{2}}^{11}+\alpha_{2}\left(z_{i} q_{2 i}\right) ; \text { for } 1 \leq i \leq 2 n-1 \text { odd }\right\} \\
& =(8 i-3)+(24 n-8 i+9) \\
& =24 n+6
\end{aligned}
$$

$$
\begin{aligned}
W_{\alpha_{2}}^{12} & =\left\{w_{\alpha_{2}}^{12}+\alpha_{2}\left(z_{i} q_{2 i-4}\right) ; \text { for } 4 \leq i \leq 2 n+2 \text { even }\right\} \\
& =(8 i-14)+(24 n-8 i+20) \\
& =24 n+6 \\
W_{\alpha_{2}}^{13} & =\left\{w_{\alpha_{2}}^{13}+\alpha_{2}\left(z_{i} q_{2 i-5}\right) ; \text { for } 4 \leq i \leq 2 n+2 \text { even }\right\} \\
& =(8 i-15)+(24 n-8 i+21) \\
& =24 n+6 \\
W_{\alpha_{2}}^{14} & =\left\{w_{\alpha_{2}}^{14}+\alpha_{2}\left(q_{i} q_{i+1}\right) ; \text { for } 1 \leq i \leq 4 n-1 \text { odd }\right\} \\
& =(4 i+3)+(24 n-4 i+3) \\
& =24 n+6 \\
W_{\alpha_{2}}^{15} & =\left\{w_{\alpha_{2}}^{15}+\alpha_{2}\left(z_{i} z_{i+1}\right) ; \text { for } 1 \leq i \leq 2 n+1 \text { odd }\right\} \\
& =(8 i-5)+(24 n-8 i+11) \\
& =24 n+6
\end{aligned}
$$

It is not difficult to see that the set $\bigcup_{t=1}^{15} W_{\alpha_{2}}^{t}=\{24 n+6,24 n+6, \ldots, 24 n+6\}$ contains an arithmetic sequence with the first term $24 n+6$ and common difference 0 . Thus $\alpha_{2}$ is a super $(24 n+6,0)$-edge-antimagic total labeling. This concludes the proof.

Proof. Case 2. for $d=2$
Label the vertices of $S t_{n}$ and label the edges of $\alpha_{3}$ for super ( $a, 2$ )-edge antimagic total labeling with the following way.

$$
\begin{aligned}
\alpha_{3}\left(x_{i} y_{i}\right) & =8 n+16 i-5, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(x_{i} z_{2 i+1}\right) & =8 n+16 i-2, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(x_{i} z_{2 i-1}\right) & =8 n+16 i-10, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(x_{i} q_{4} i\right) & =8 n+16 i-3, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(x_{i} q_{4 i-1}\right) & =8 n+16 i-4, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(y_{i} z_{2 i+2}\right) & =8 n+16 i \text {, for } 1 \leq i \leq n \\
\alpha_{3}\left(y_{i} z_{2 i}\right) & =8 n+16 i-8, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(y_{i} q_{4 i-2}\right) & =8 n+16 i-6, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(y_{i} q_{4 i-3}\right) & =8 n+16 i-7, \text { for } 1 \leq i \leq n \\
\alpha_{3}\left(z_{i} q_{2 i-1}\right) & =8 n+8 i-4, \text { for } 1 \leq i \leq n, \text { odd } \\
\alpha_{3}\left(z_{i} q_{2 i}\right) & =8 n+8 i-3, \text { for } 1 \leq i \leq n, \text { odd } \\
\alpha_{3}\left(z_{i} q_{2 i-4}\right) & =8 n+8 i-14, \text { for } 4 \leq i \leq n, \text { even } \\
\alpha_{3}\left(z_{i} q_{2 i-5}\right) & =8 n+8 i-15, \text { for } 4 \leq i \leq n, \text { even } \\
\alpha_{3}\left(q_{i} q_{i+1}\right) & =8 n+4 i+3, \text { for } 1 \leq i \leq 2 n+2, \text { odd } \\
\alpha_{3}\left(z_{i} z_{i+1}\right) & =8 n+8 i-5, \text { for } 1 \leq i \leq 2 n+2, \text { odd }
\end{aligned}
$$

The total labeling $\alpha_{3}$ is a bijective function. The set $\{1,2,3, \ldots, 24 n+3\}$. The edgeweights of $S t_{n}$, under the labeling $\alpha_{3}$, constitute the sets:

$$
\begin{gathered}
W_{\alpha_{3}}^{1}\left(x_{i} y_{i}\right)=8 n+32 i-10, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{2}\left(x_{i} z_{2 i+1}\right)=8 n+32 i-4, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{3}\left(x_{i} z_{2 i-1}\right)=8 n+32 i-20, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{4}\left(x_{i} q_{4 i}\right)=8 n+32 i-6, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{5}\left(x_{i} q_{4 i-1}\right)=8 n+32 i-8, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{6}\left(y_{i} z_{2 i+2}\right)=8 n+32 i, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{7}\left(y_{i} z_{2 i}\right)=8 n+32 i-16, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{8}\left(y_{i} q_{4 i-2}\right)=8 n+32 i-12, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{9}\left(y_{i} q_{4 i-3}\right)=8 n+32 i-14, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n} \\
W_{\alpha_{3}}^{10}\left(z_{i} q_{2 i-1}\right)=8 n+16 i-8, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n}, \text { odd } \\
W_{\alpha_{3}}^{11}\left(z_{i} q_{2 i}\right)=8 n+16 i-6, \text { for } 1 \leq \mathrm{i} \leq \mathrm{n}, \text { odd } \\
W_{\alpha_{3}}^{12}\left(z_{i} q_{2 i-4}\right)=8 n+16 i-28, \text { for } 4 \leq \mathrm{i} \leq \mathrm{n}, \text { even } \\
W_{\alpha_{3}}^{13}\left(z_{i} q_{2 i-5}\right)=8 n+16 i-30, \text { for } 4 \leq \mathrm{i} \leq \mathrm{n}, \text { even } \\
W_{\alpha_{3}}^{14}\left(q_{i} q_{i+1}\right)=8 n+8 i+6, \text { for } 1 \leq \mathrm{i} \leq 2 \mathrm{n}+2, \text { odd } \\
W_{\alpha_{3}}^{15}\left(z_{i} z_{i+1}\right)=8 n+16 i-10, \text { for } 1 \leq \mathrm{i} \leq 2 \mathrm{n}+2, \text { odd }
\end{gathered}
$$

We can found the total labeling $W_{\alpha_{3}}$ with summing $w_{\alpha_{1}}=w_{\alpha_{3}}$ with edge label $\alpha_{3}$. It is not difficult to see that the set $\bigcup_{t=1}^{15} W_{\alpha_{3}}^{t}=\{8 n+6,8 n+8,8 n+10 \ldots, 40 n+6\}$ contains an arithmetic sequence with the first term $8 n+6$ and common difference 0 . Thus $\alpha_{3}$ is a super $(8 n+6,2)$-edge-antimagic total labeling. This concludes the proof.

## 5 Conclusion

1. There are a super $(a, d)$-edge-antimagic total labeling of graph $S t_{n}$, if $n \geq 2$ with $d \in\{0,1,2\}$.

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